# Recent Advances and Trends in Applied Algebraic Topology

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# Outline

# The History of the field

| | |
|---|---|
| 1845 | Kirchhoff formulates "homological" circuit laws. |
| 1895 | Analysis Situs by Henri Poincaré defines homology. |
| 1925-28 | Noether, Mayer & Vietoris categorify homology. |
| 1993 | Size theory |
| 1996 | Incremental Betti numbers |
| 2002 | Persistent homology |
| 2005 | Algebraic persistent homology |
| 2007 | Stability theorems |
| 2007 | Extended persistence |
| 2007 | Multi-dimensional persistence |
| 2008 | Zig-zag persistence |
| 2009 | Algebraic stability |
| 2009 | Persistent cohomology |
| 2010 | Well groups |
| 2011 | Dualities (absolute/relative (co)homology share barcodes) |
| 2012 | Categorification of persistence |

# Current trends

Formalization: $\Bbbk[t]$-modules. Order representations. Categorical diagrams.

Enlargement: more topological techniques are introduced into the field.

Speed up: faster computation means larger problems; parallelization, simplifying pre-processing.

More application fields: information flow networks, ethology, phylogenetics, sports, ...

# Outline

# Paweł Dłotko

## Cohomology in electromagnetic modeling.

- Based on reformulation of Maxwell's laws in a discrete setting.
- Local version of laws (Ampere, Faraday,...) formulated on primal/dual cell complex by using (co)boundary operator.
- We impose *global* version of the laws by adding cohomological information.
- We show that first cohomology basis of the insulating region is the missing d.o.f. that make the computations consistent.
- We provide efficient software to compute cohomology groups & generators.

# Paweł Dłotko

## Homology for regular CW-complexes.

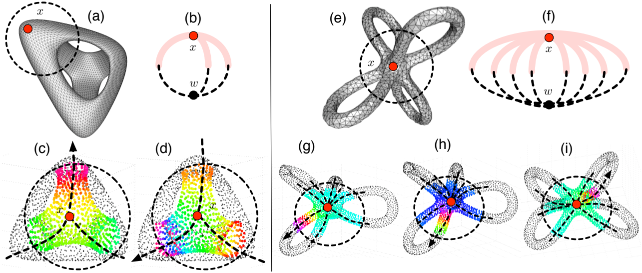
- Usually (co)homology of simplicial or cubical complexes is considered in applied science.
- We have introduced algorithm providing (co)homology of any regular CW complex.
- Gain – when the data do not fit the simplicial or cubical structure.
- *Input:* list of faces of every cell. *Output:* (co)homology. Incidences of cells computed by the algorithm.
- *Example:* homology of non–regular cubical grid which efficiently approximates homology of nodal domains of functions.
- Technique used in statistical simulation of spinodal decomposition in alloys, providing orders of magnitude better performance.

# Bei Wang, University of Utah

Local homology measures behavior very close to a point. Everything outside a small circle collapses to a single point.

*n*-dimensional manifolds turn into *n*-spheres; we can use local homology to measure dimensionality.

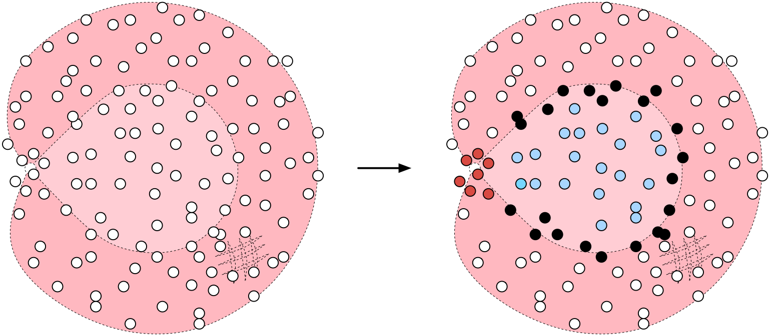
More importantly, singular or branching behavior corresponds to wedges of spheres.



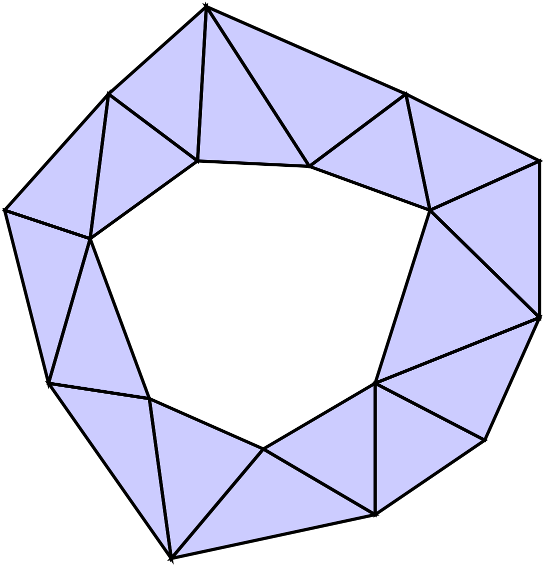
# Bei Wang, University of Utah

## Results

- Can classify strata in stratified spaces; turn up inter alia when analyzing conformation spaces in robotics or in chemistry.
- Can classify branching points in self-intersecting time series data.



# Vidit Nanda, Rutgers



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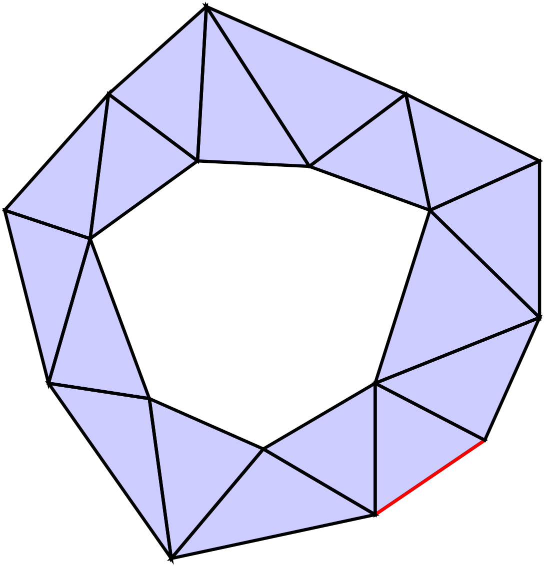

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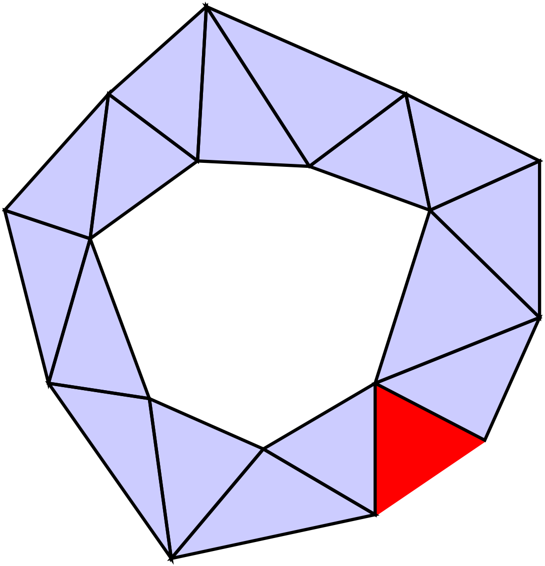

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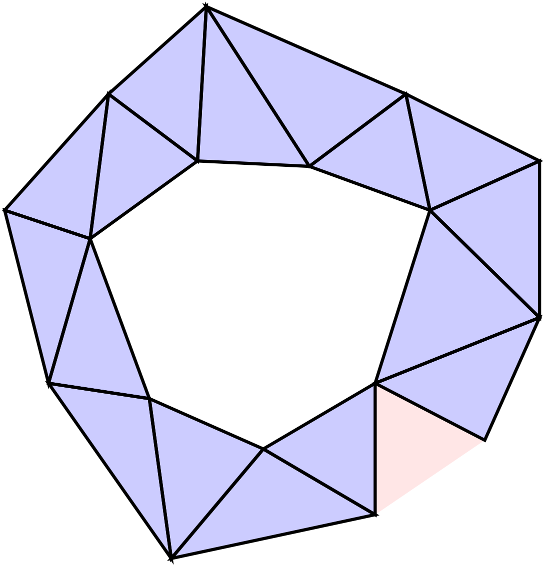

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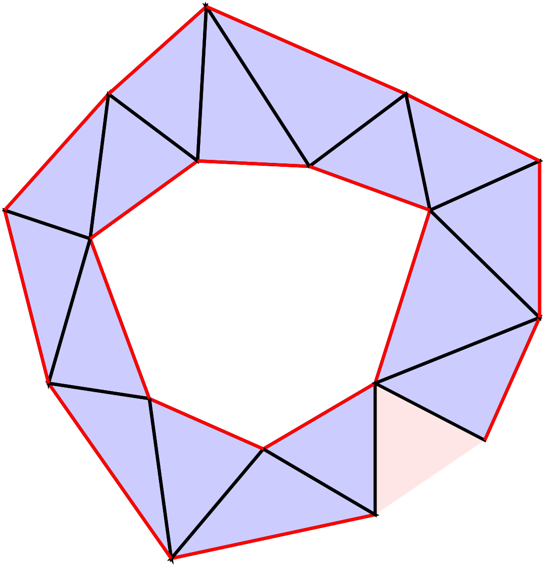

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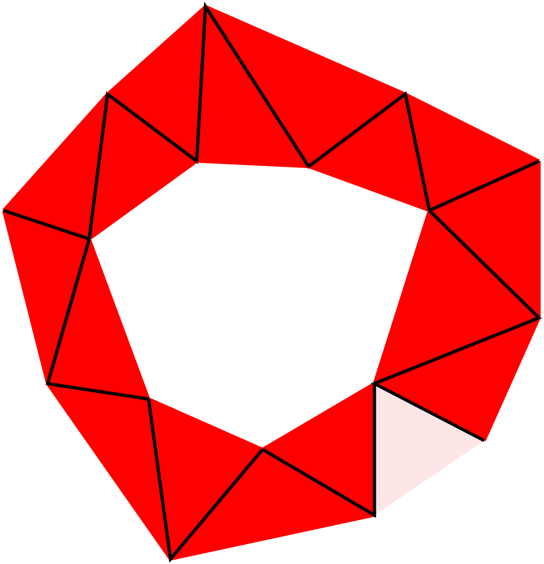

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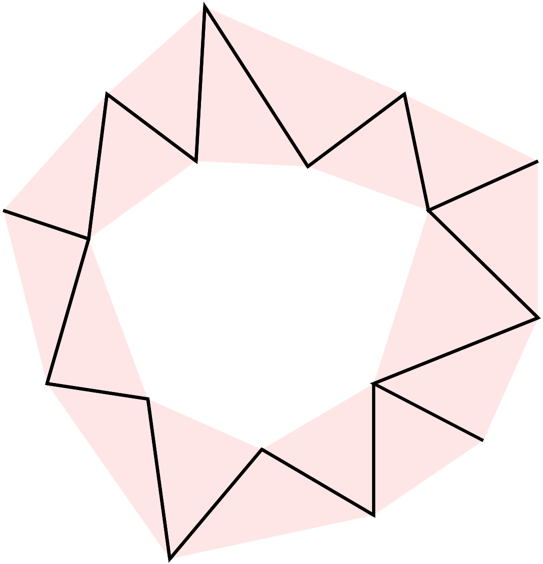

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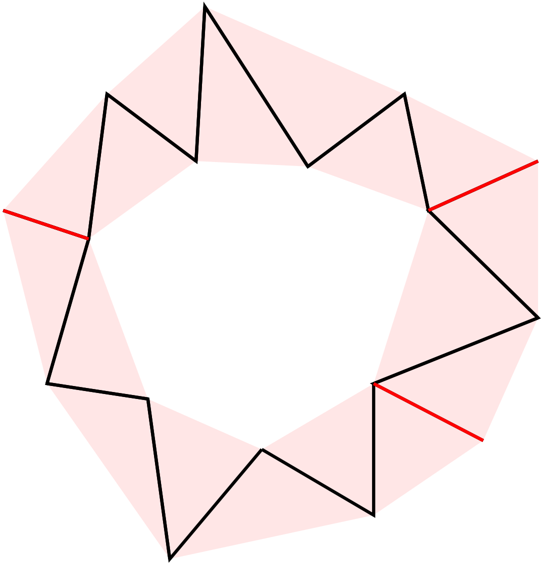

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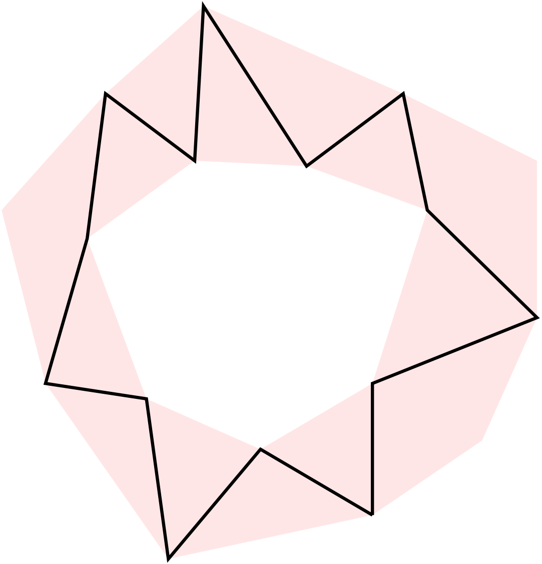


# Vidit Nanda, Rutgers

## Main result

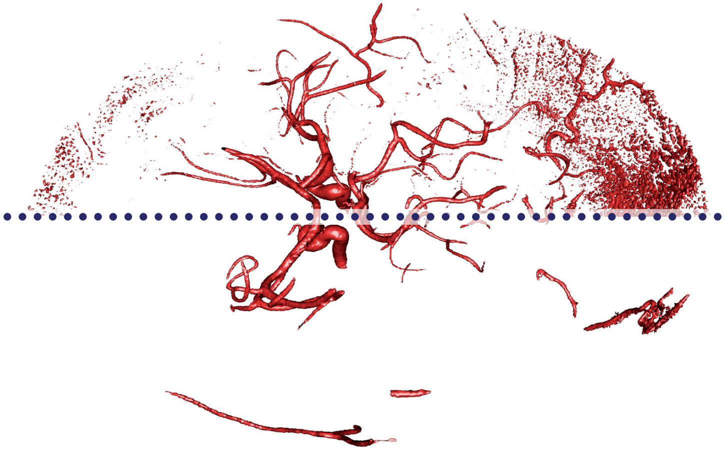
Discrete Morse theory is compatible with persistent homology. A filtered complex contracts to a filtered complex. This speeds up computations in applied algebraic topology.

## Implementation available

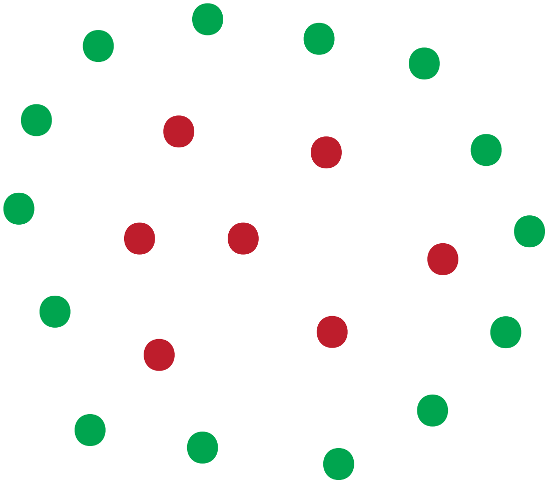
Part of the Perseus project.
Available at `http://www.math.rutgers.edu/~vidit/perseus.html`

# Ulrich Bauer, IST Austria

Uses discrete morse theory for topological simplification.



# Elizabeth Munch, Duke



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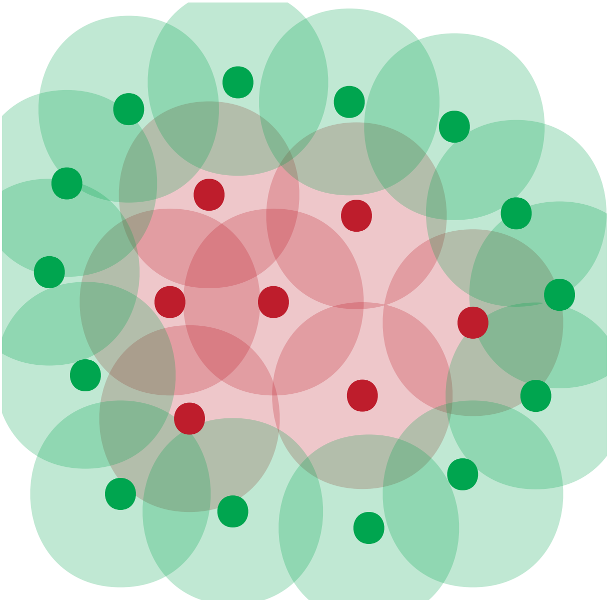

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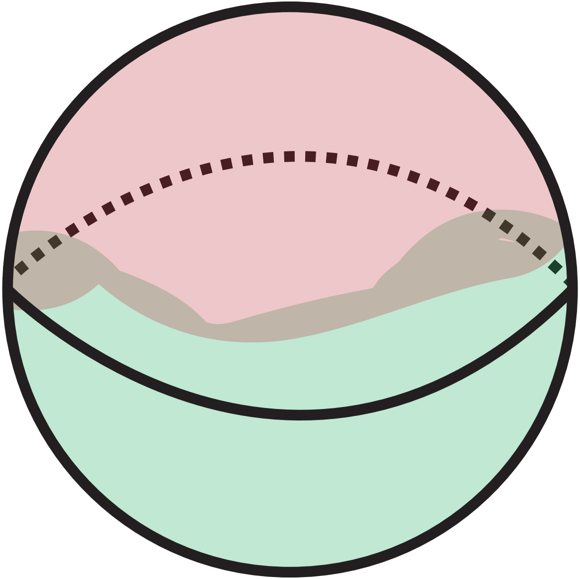

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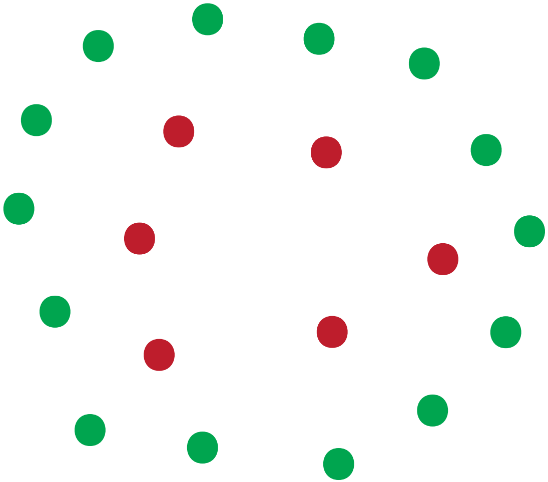

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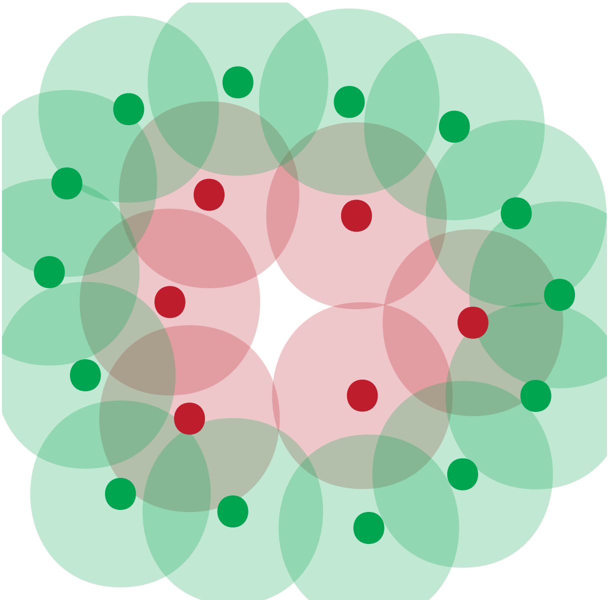

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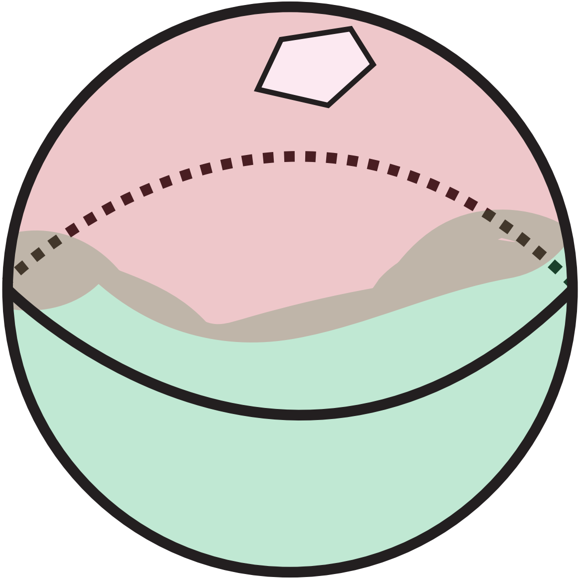


# Elizabeth Munch, Duke

## Results

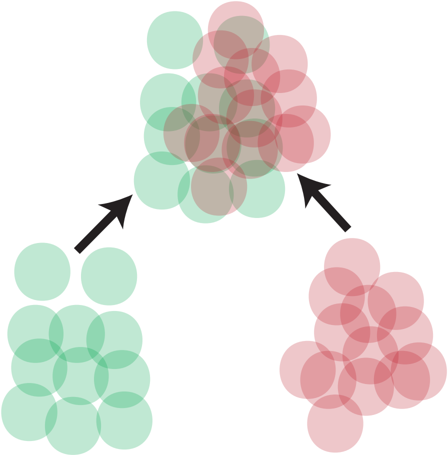
Suppose sensors fail with some probability. Then:

- Computing the probability of failure of the $H_2$ criterion is $\#P$-hard.
- If imminent failure can be detected by a monitoring system, an algorithm is provided.

# Jennifer Gamble, NCSU

A sensor network that varies through time, where sensors are mobile or transient, poses additional challenges.

Zig-zag homology provides the tools needed to track coverage holes across several snapshots.



# Hungry for more?

## Survey papers

*Topology and Data* by Gunnar Carlsson
*Barcodes: the persistent topology of data* by Robert Ghrist

## Topic conferences

ATMCS – biennial conference on applied and computational algebraic topology
ACM SoCG – computational geometry conference with strong computational topology sessions

## Aggregating websites

`http://comptop.stanford.edu` – Homepage of Gunnar Carlsson's workgroup. Contains conference and preprint aggregation.
`http://www.appliedtopology.org` – yet to be launched; coordinating webpage for the community, centered around ATMCS.