Beauty in Mathematics
Mathematics in Beauty

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- Beauty in Mathematics
  Elegance in Ideas and Connections

- Mathematics in Beauty
  Art that draws on, Art that illustrates Mathematics

- Mathematical Beauty
  The Maker Revolution

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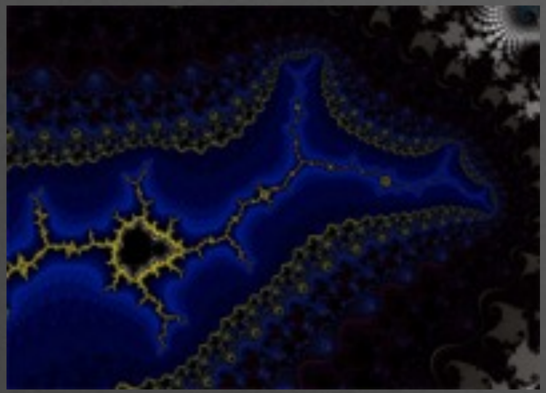
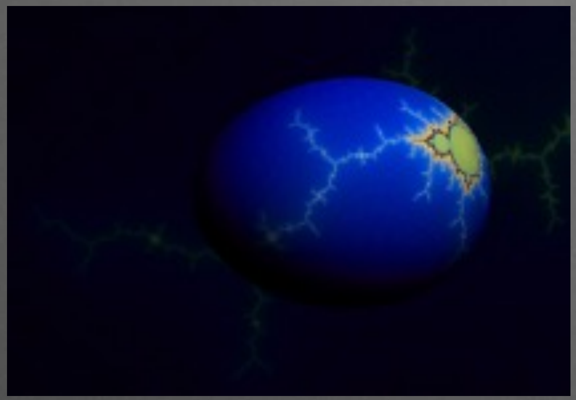
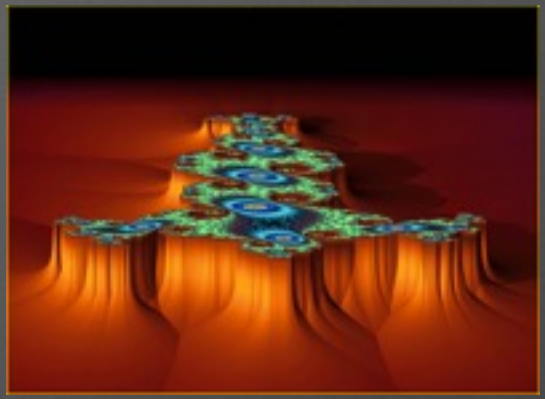
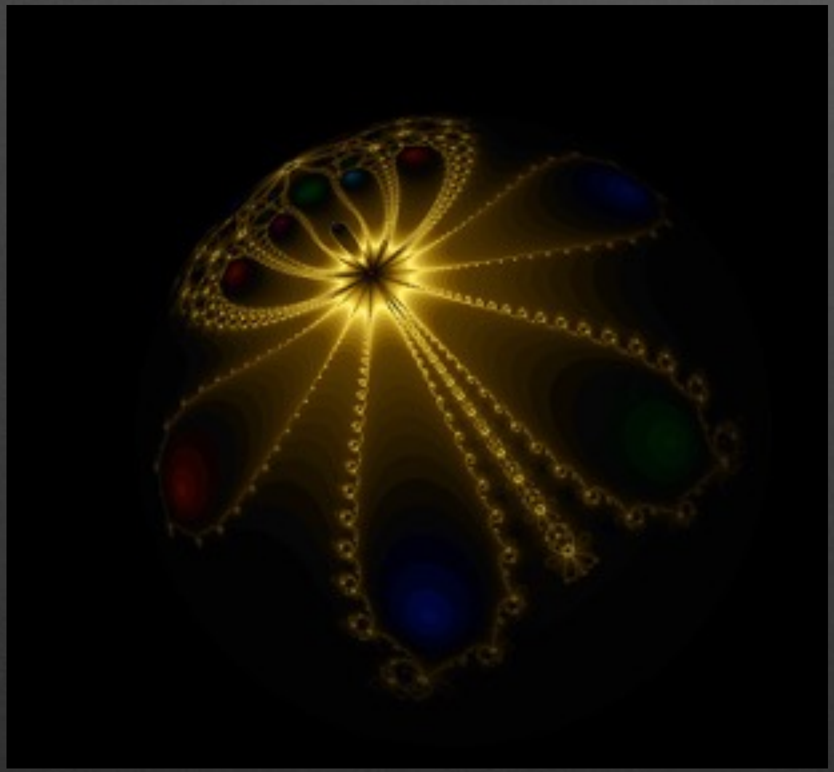
Accidental Beauty: Fractals, Patterns


Abstract Beauty:
The aesthetics of *ideas*

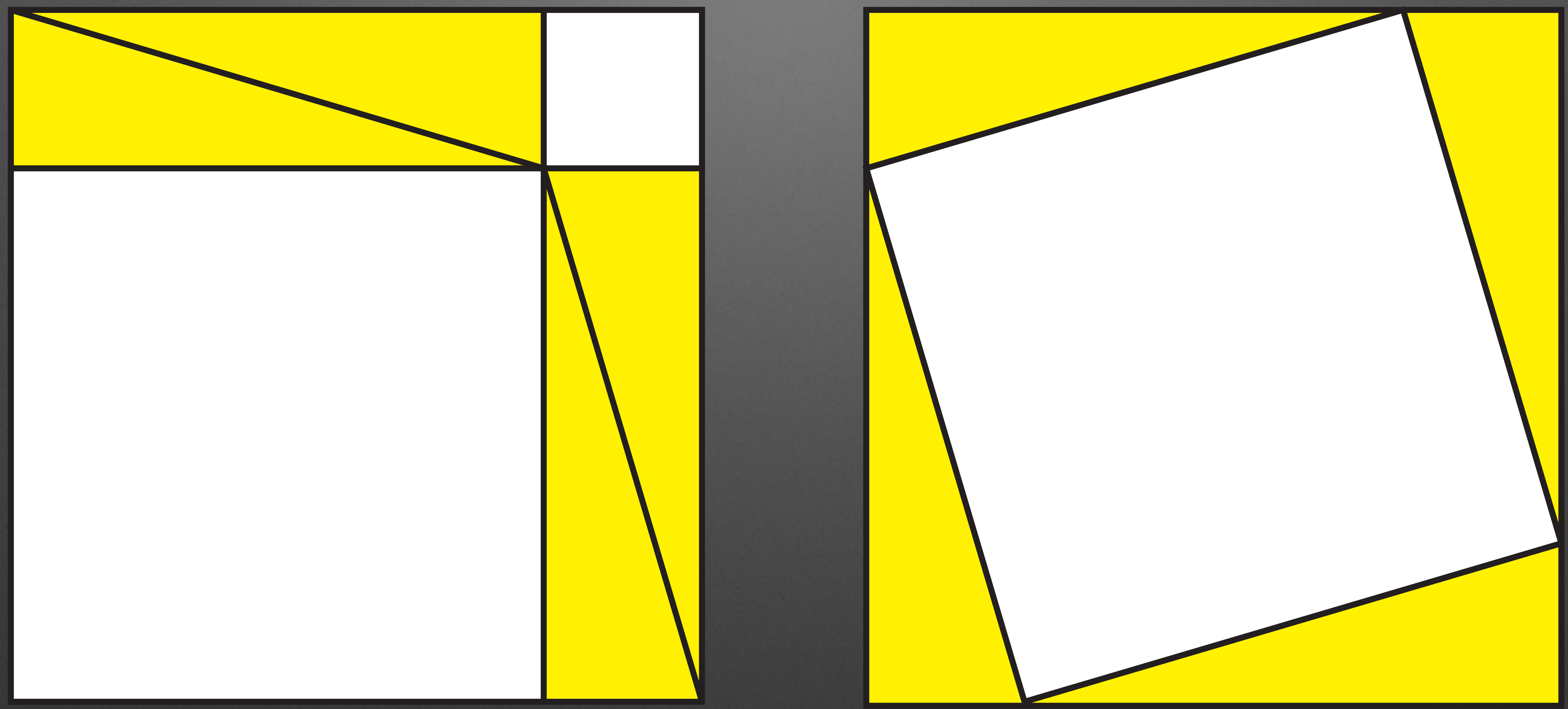
- Core aspect of mathematical beauty: **Elegance**

- Hardy:
  Inevitability, Unexpectedness and Economy

- Erdős:
  God keeps **The Book**, containing the most elegant proof of each mathematical theorem.

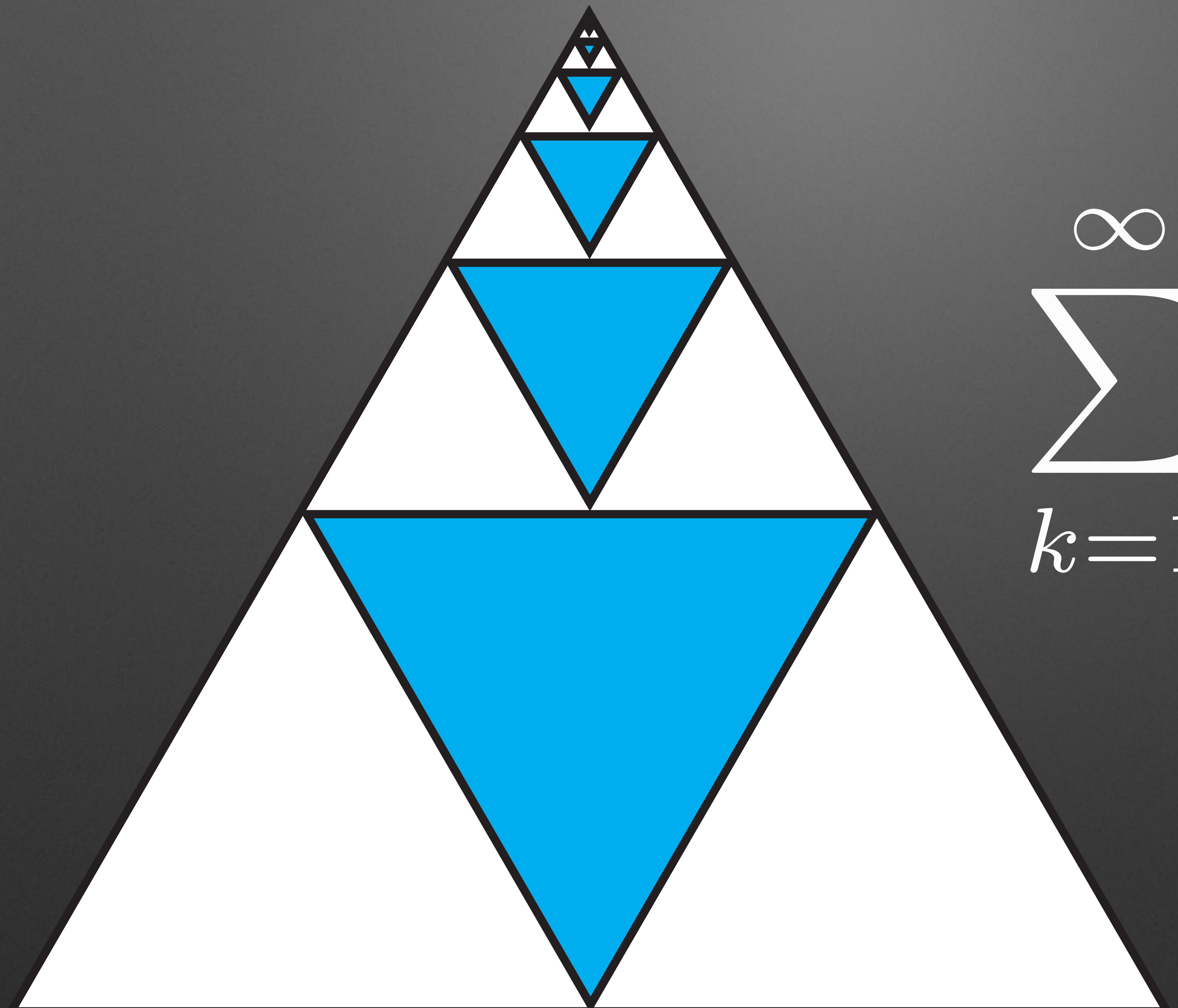
  You don't have to believe in God, but you should believe in The Book.

Proofs without words



$$a^2 + b^2 = c^2$$

Proofs without words



$$\sum_{k=1}^{\infty} \frac{1}{4^k} = \frac{1}{3}$$

Beautiful Proofs

- **Theorem:** There are infinitely many prime numbers
- **Proof:** Suppose there were finitely many, $p_1, \ldots, p_k$. Multiply them all together, add 1 to form a new number $N = p_1 \ldots p_k + 1$.

  N is not divisible by any one of the p_j - it gives a residue of 1. Either N is prime, or it is a product of primes not in the list.

  Hence no finite list of primes can be complete.

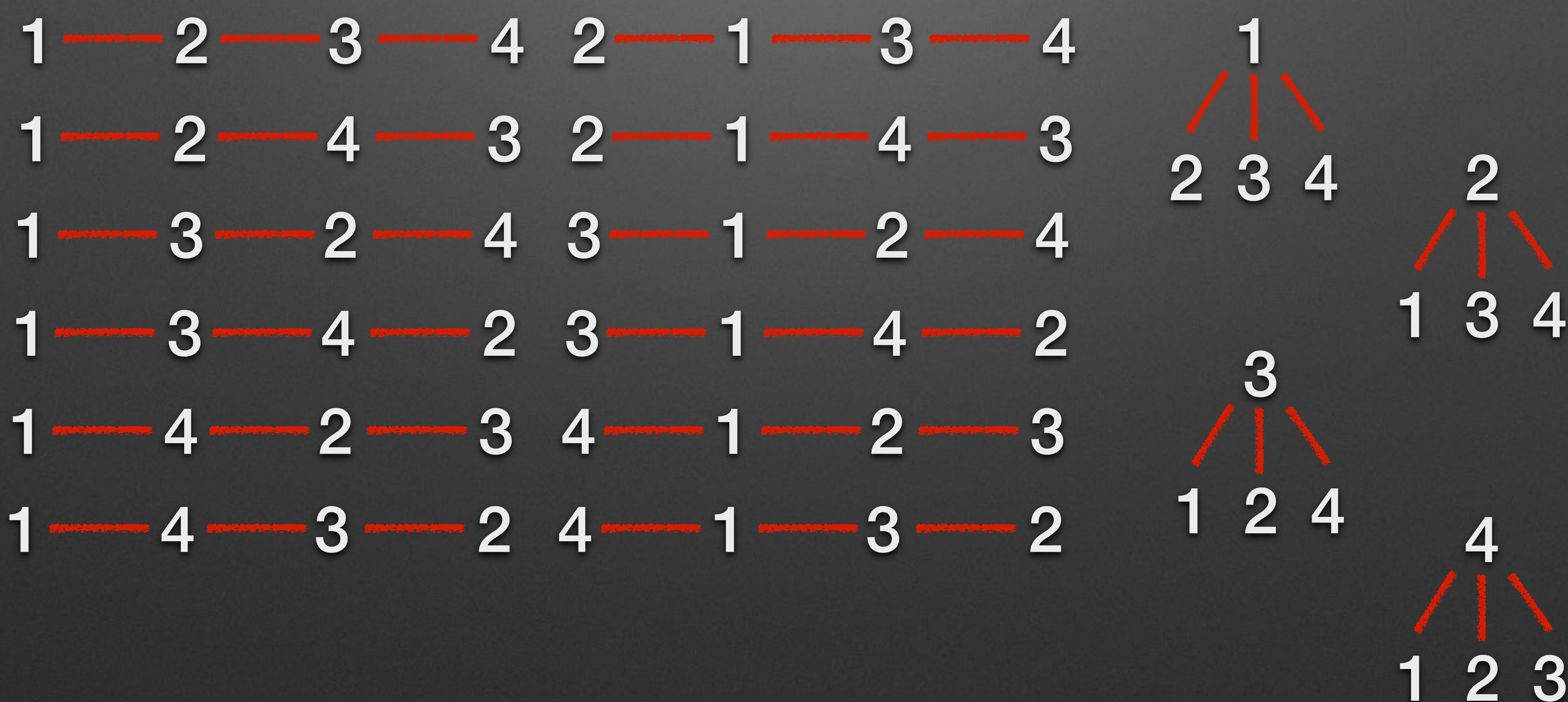
Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.

2 — 1 — 3 1 — 2 — 3 1 — 3 — 2

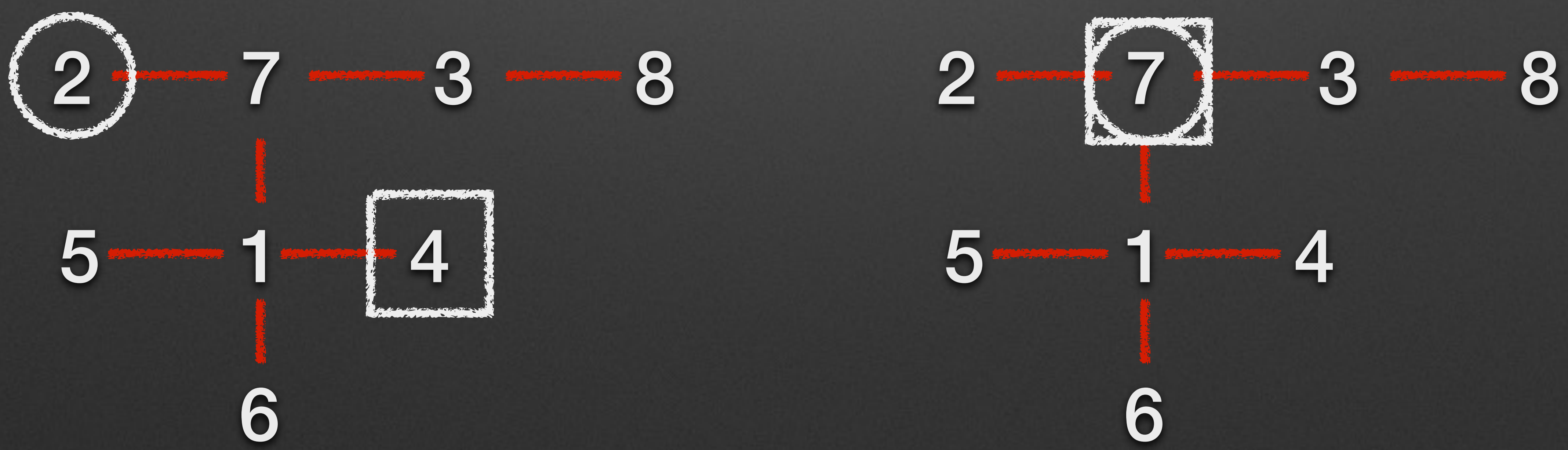
Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.



Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof:** First, consider trees with two vertices marked. The marked vertices can be picked in n ways each - so the number of trees = number of bi-marked trees / n^2.



Beautiful Proofs

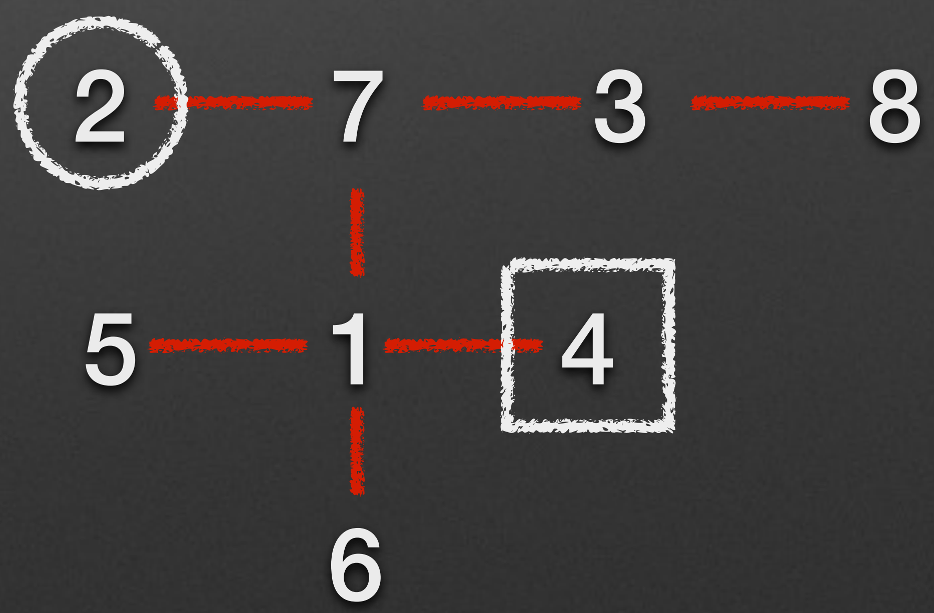
- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** If there are n^n bi-marked trees, then the theorem is proven.

  One thing there are n^n of is functions $\{1,\ldots,n\} \to \{1,\ldots,n\}$.

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
2	7	7	1	1	1	4	3

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** From a bi-marked tree we build a function. First stretch the tree by the markings.



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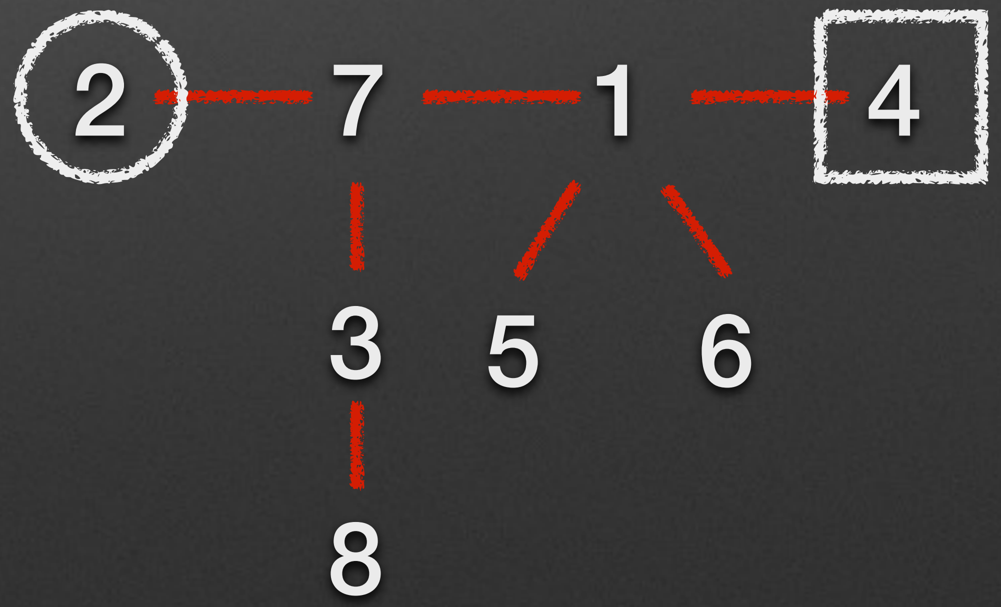

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** From a bi-marked tree we build a function.

  Next, fill in the function for the path between the marks by just reading off the order in the path

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
2	7	?	1	?	?	4	?

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** From a bi-marked tree we build a function.

  Finally, let the tails flow towards the bi-marked path.

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** Any bi-marked tree creates a different function.

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
2	7	7	1	1	1	4	3

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** Any function creates a different bimarked tree: pick out *cycles* in the function: the lower row forms the marked path. The remaining parts attach to the path as the function specifies.

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
2	4	2	5	1	6	4	5

Beautiful Proofs

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1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
2	4	2	5	1	6	8	5

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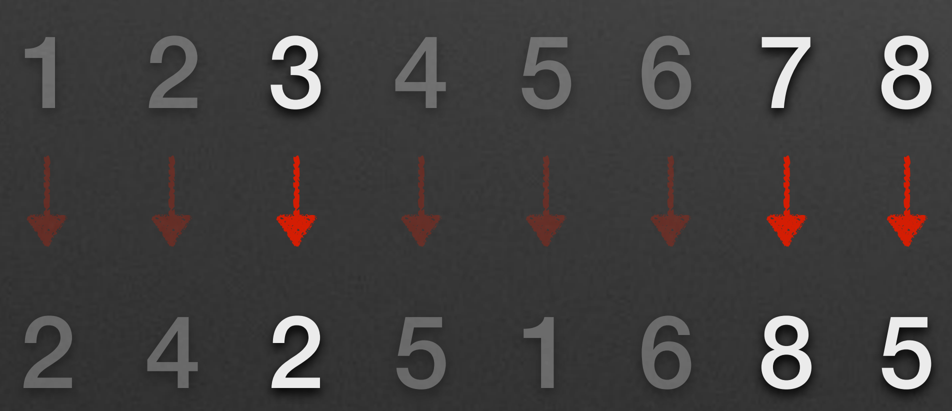

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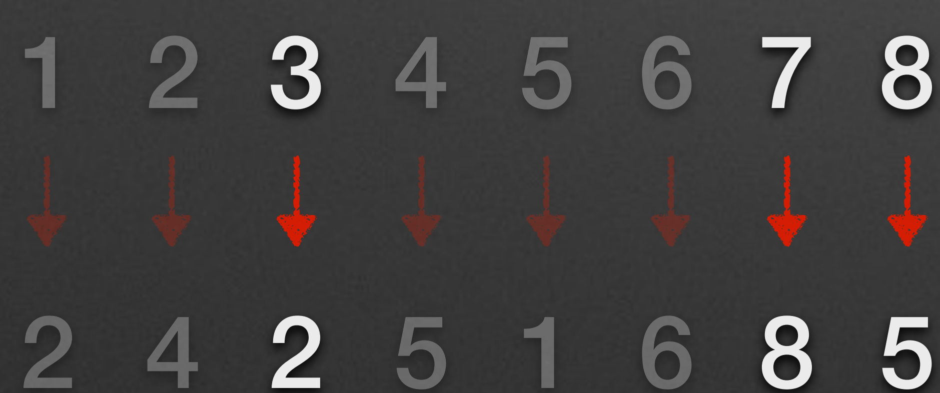

Beautiful Proofs

- **Theorem:** There are exactly n^{n-2} trees with vertices 1, …, n.
- **Proof…:** In conclusion:
  Every bi-marked tree produces a function
  Every function produces a bi-marked tree.

  So there are n^n bi-marked trees.

  So there are n^{n-2} trees.

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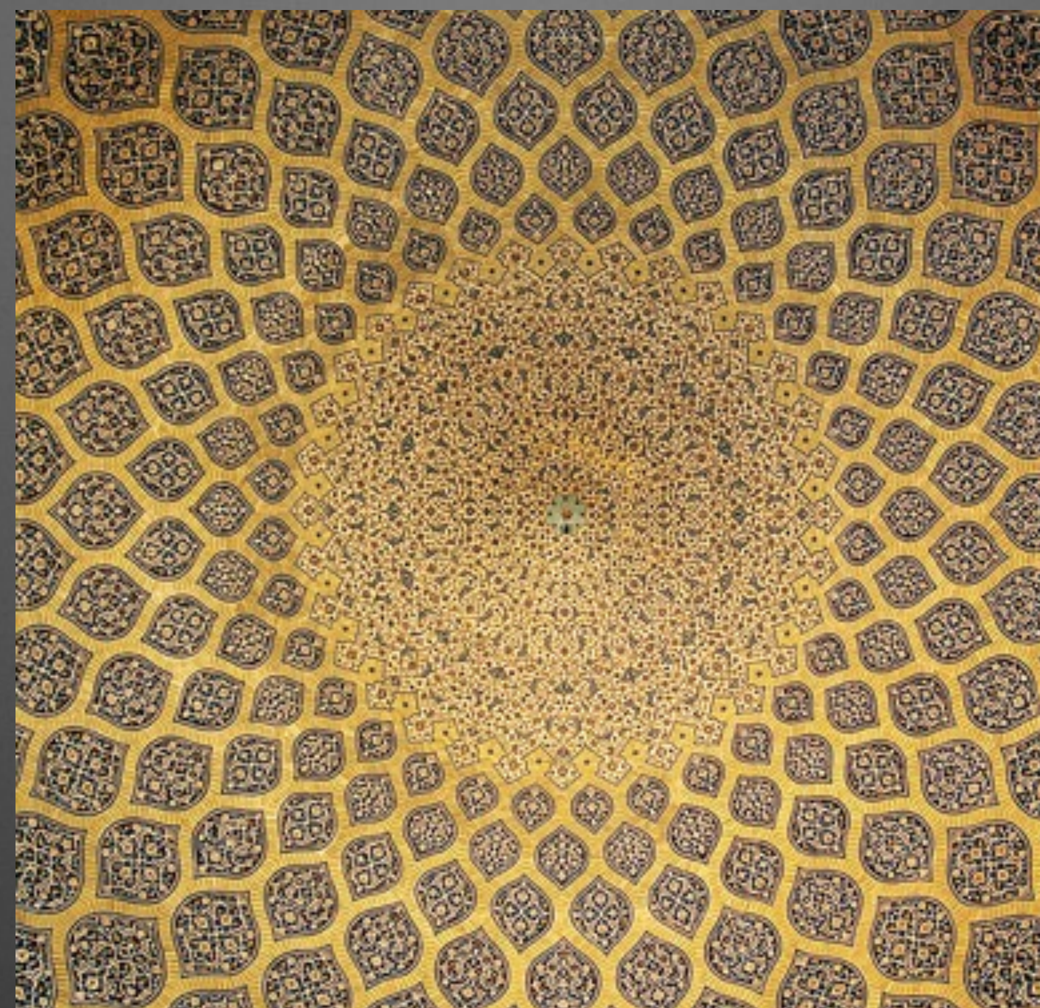
Albrecht Dürer (1514)

Anamorphosis
Hans Holbein (1533)

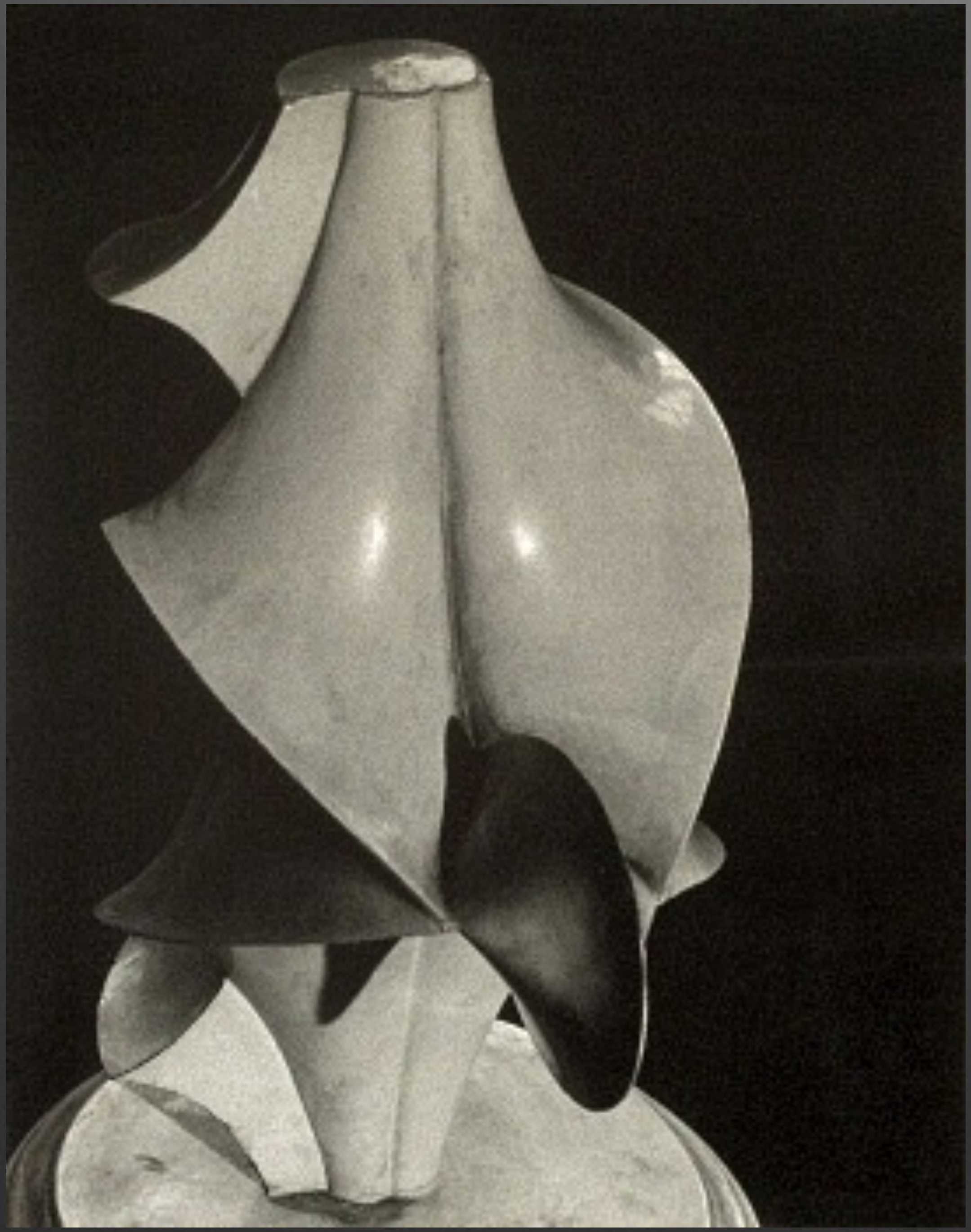
Anamorphosis
Hans Holbein (1533)

Symmetries



It has been claimed that all 12 possible wallpaper symmetry structures occur in the ornaments at Alhambra.

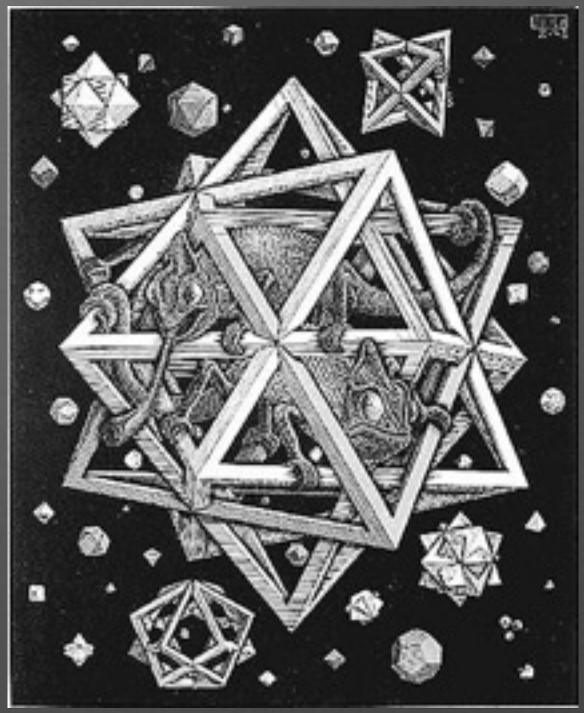
Mathematical objects as art

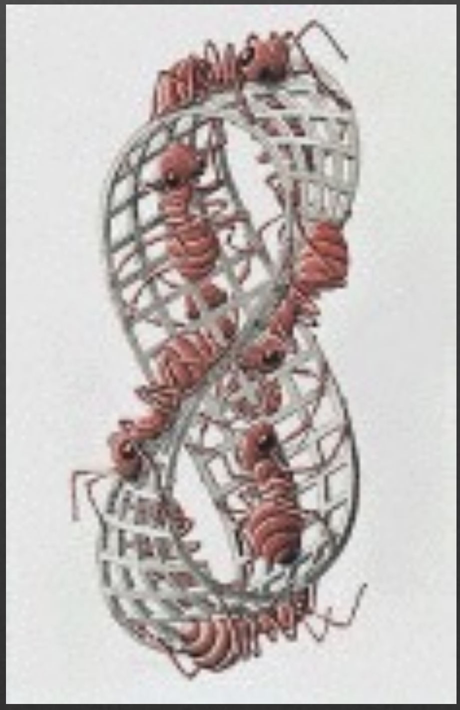


Man Ray:
Objet mathematique

Model of an Enneper surface found at Institute Henri Poincaré

MC Escher Mathematical Shapes





MC Escher
Symmetries and Tilings

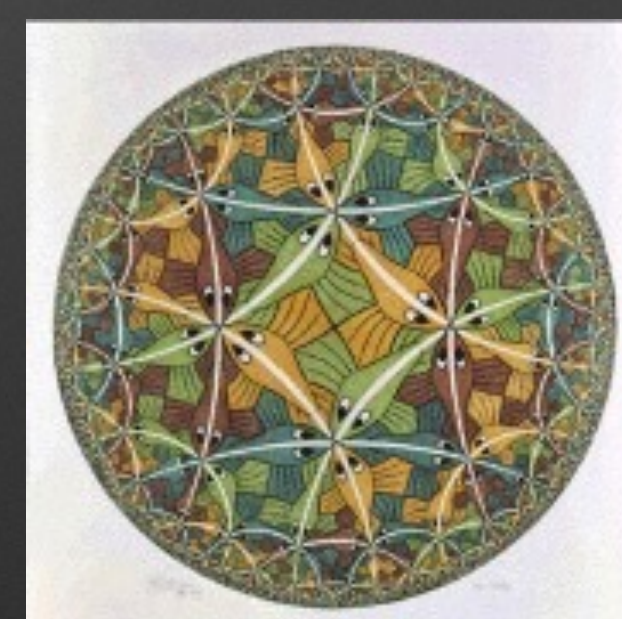









MC Escher
Non-Flat Geometry



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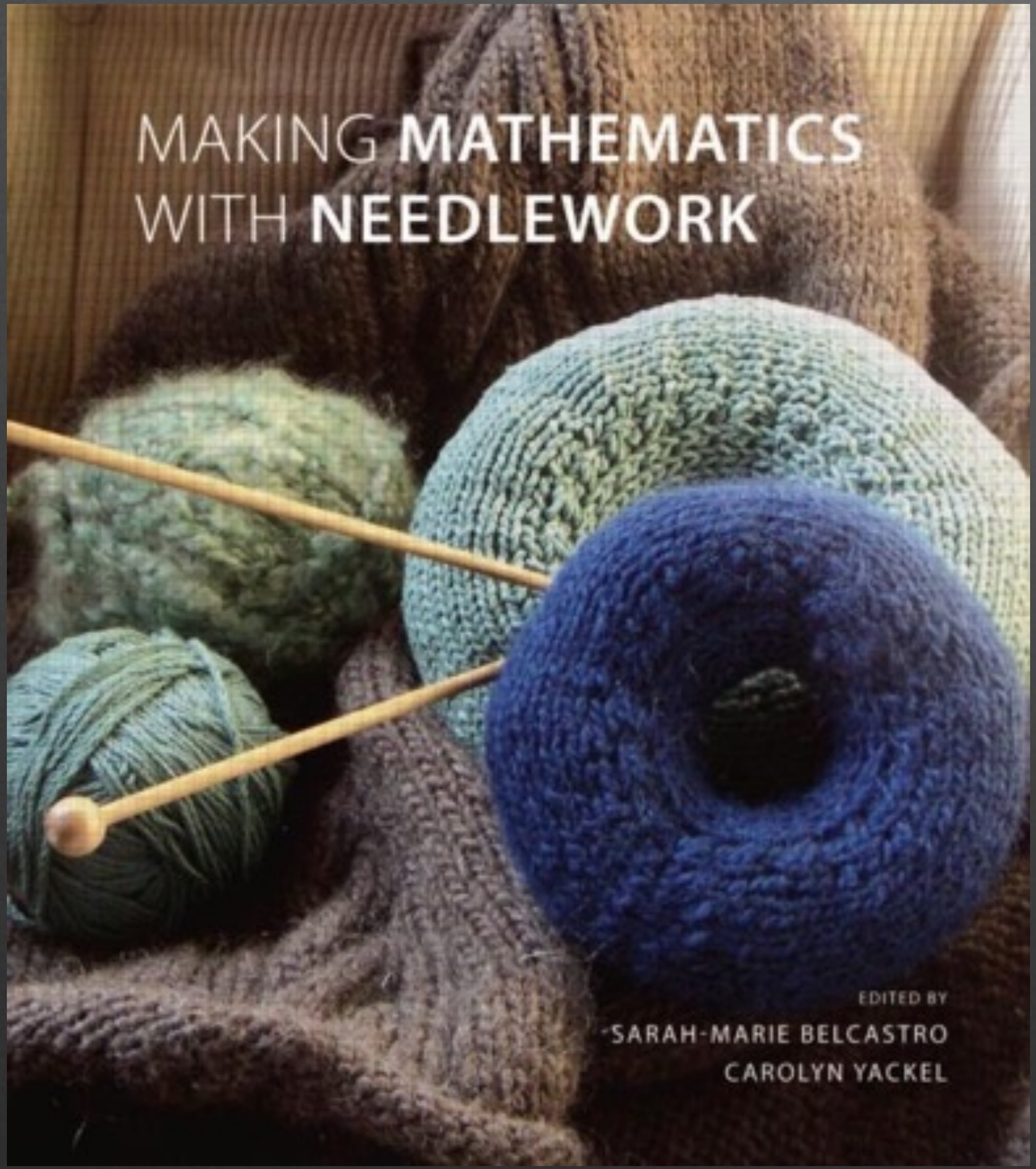
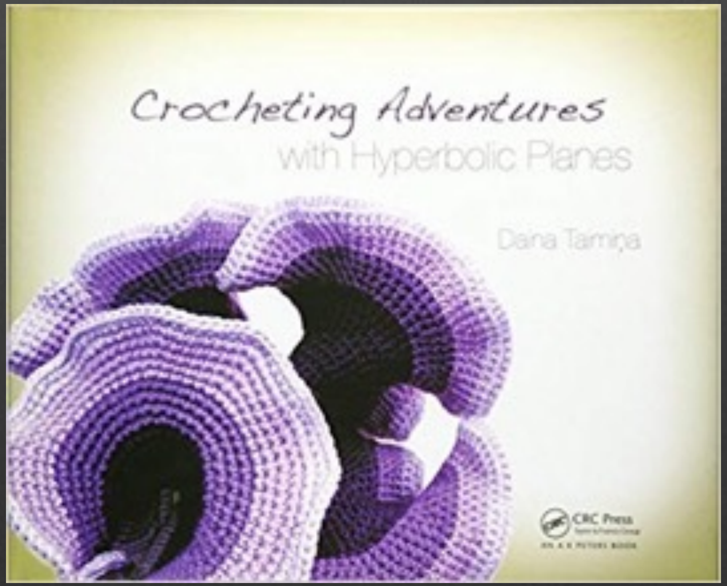
Mathematics in Crafts

Every Year, the Mathematical Knitting Circle convenes





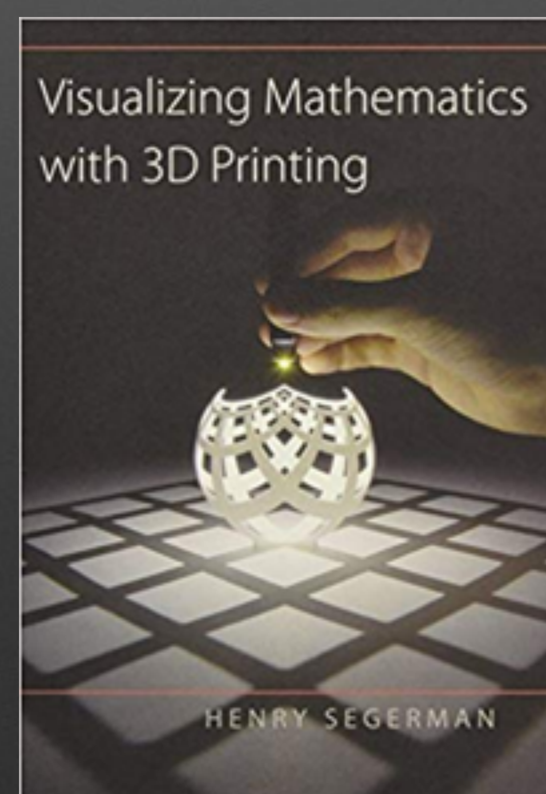
Computerized Manufacturing The Maker Revolution

- Small scale manufacturing tools are now available at consumer prices
- Laser cutters, 3d printers, CNC mills, Vacuum formers
- Generates an entire genre of algorithmic, reified mathematical art

Cutting and Etching

3d printing

Bathsheba Grossman



Algorithmic Knitting

Fabienne Serriere